Grade 6 Math Circles

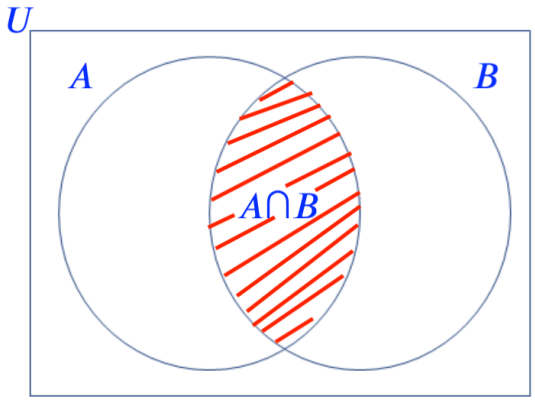
November 8/9/10, 2022

Introduction to Set Theory - Problem Set

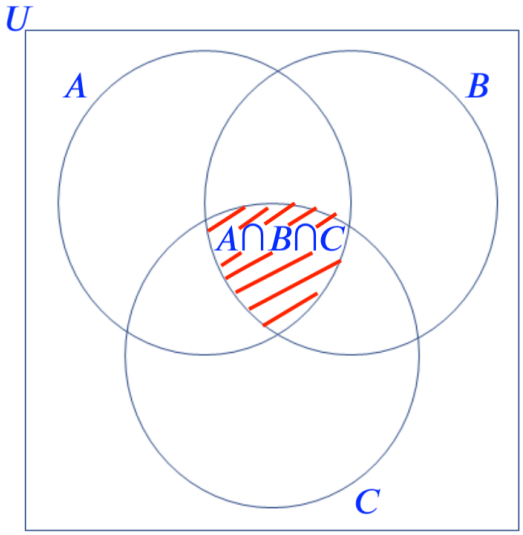
1. Rewrite these set definitions by listing the elements in the set.
   (a) $A = \{x \mid x \text{ is an upper-case letter}\}$.
   (b) $B = \{x \mid x \text{ is a Canadian province or territory}\}$.
   (c) $C = \{x \mid x \text{ is an odd positive integer}\}$.
   (d) $D = \{x \mid x \text{ is a prime number greater than } 2\}$.
   (e) $E = \{x \mid x \text{ is a negative number greater than } 0\}$.
   (f) $F = \{x \mid x \text{ is the empty set}\}$.
2. Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universal set.
   Let $A = \{0, 1, 2, 8, 9\}$ and $B = \{1, 3, 5, 7, 9\}$. Find the following sets by listing all the elements.
   (a) $A \cup B$
   (b) $A \cap B$
   (c) A^C
   (d) B^C
   (e) $A \setminus B$
   (f) $B \setminus A$
   (g) $(A \cup B)^C$
3. Let $A = \{x \mid x \text{ is a real number greater than } 0\}$.
   Let $B = \{x \mid x \text{ is a real number less than or equal to } 7\}$.
   Write the set definitions of the following set by using the elementhood test.
   (a) $A \cap B$
   (b) $A \cup B$
   (c) $B \cap \mathbb{Z}$
   (d) $A \cup \mathbb{N}$

4. Find the cardinality of the following sets
   (a) $\{101, 102, 103, 104\}$
   (b) {dog, cat}
   (c) $\big\{\{a, b\}, b, \{a, b, c\}\big\}$
   (d) $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$
   (e) $\{x \in \mathbb{N} \mid x \leq 10\}$ (or $\{x \mid x$ is a natural number less than or equal to $10\}$)
   (f) $\{x \mid x \in \mathbb{W} \setminus \mathbb{N}\}$
   (g) $\{x \mid x$ is a prime number$\} \cap \{x \mid x$ is an even positive integer$\}$
   (h) $\mathbb{Q} \cap \varnothing$
5. Let A and B represent sets.
   (a) Is it always the case that $(A \cap B) \subseteq (A \cup B)$? Why or why not?
   (b) Is it always the case that $(A \cup B) \subseteq (A \cap B)$? Why or why not?
   (c) If you have $(A \cap B) \subseteq (A \cup B)$ and $(A \cup B) \subseteq (A \cap B)$, what do you know about the two sets A and B?
6. Let A and B represent sets.
   (a) Is it always the case that $(A \setminus B) \subseteq (A \cap B^C)$? Why or why not?
   (b) Is it always the case that $(A \cap B^C) \subseteq (A \setminus B)$? Why or why not?
   (c) What can you conclude from (a) and (b)?
7. Let A and B represent sets.
   (a) Is it always the case that $((A \cup B) \setminus B) \subseteq A$? Why or why not?
   (b) Give an example of two sets A and B for which $(A \cup B) \setminus B \neq A$.

8. Below is the Venn diagram for 2 sets, A and B. Notice that the overlapping part is the intersection of A and B. Think about what part of the diagram represents the union of A and B.



Below is the Venn diagram for 3 sets, A, B, and C. Notice $A \cap B \cap C$ is marked where three circles (sets) are overlapping. Think about what parts of the diagram represent $(A \cap B), (B \cap C), (A \cap C), (A \cup B), (B \cup C), (A \cup C), (A \cup B \cup C)$, etc.



Now, do you think it is possible to draw a Venn diagram for 4 sets, A, B, C, and D? If so, how?

9. **Introduction to Power Set**

Power Set

Given a set A, we define the **power set** of A to be the set

$$\mathbb{P}(A) = \{X \mid X \subseteq A\}.$$

In words, the power set of A, $\mathbb{P}(A)$, is a collection of all subsets of A including $\varnothing$ and A itself.

Let $A = \{0, 1\}, B = \{0, 1, 2\}$, and $C = \{0, 1, 2, 3\}$.

(a) Find $\mathbb{P}(A), \mathbb{P}(B)$, and $\mathbb{P}(C)$.

(b) Find the cardinality of A, B, and C.

(c) Find the cardinality of $\mathbb{P}(A), \mathbb{P}(B)$, and $\mathbb{P}(C)$.

(d) What observation can you make with what you have found in (b) and (c)?